A Workshop Based Approach to Calculus Pedagogy

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1 Introduction

The traditional classroom environment leaves little opportunity to encourage students to undertake more challenging multi–part problems. Further, Calculus homework exercises are primarily concerned with developing the skill sets of the students to achieve a level of comfort and proficiency with the procedures and a basic understanding of the concepts learned in the classroom.

Most students feel that if they do not see the answer after five or six minutes of thought, the problem is "too hard" or "undoable". This is troublesome for a variety of reasons. Assuming that Calculus is a prerequisite for a science class (this accounts for nearly 80% of Calculus students at CSU, Monterey Bay), the students will be using the mathematics skills learned in the classroom not to solve integrals or take derivatives for the sake of finding an answer, but will be incorporating this knowledge with their scientific skill set to aid them in their research, in other words as part of a much larger problem requiring many hours of work.

The impetus behind the Calculus Workshop was threefold: to address the issues discussed above, to encourage students traditionally under–represented in mathematics to become more involved and possibly major or minor in mathematics or science and to foster an appreciation of mathematics and problem solving. This paper will discuss my methods of facilitating the Workshop and observations of students' activities using one sample problem I found to be well suited for this type of environment as a jumping off point.

2 Project Details

Initially, I received partial funding from an Alliance for Minority Participation NSF grant as well as a small grant from the CSUMB Foundation to explore and develop this model. A group of students meets with me weekly to work on challenging problems in a group setting. Participation is completely voluntary and the number of attendees has varied by semester, from a high of twenty-two students (roughly five groups of students) to a low of eight (two groups of students). Currently, I am leading the sixth cohort of students participating in the Calculus Workshop. This research will analyze the effects these workshops have on students understanding of mathematical concepts, in particular, those related to Calculus.

In studying the high failure rates of African-American students at University of California, Berkeley during the late 1970's and 1980's, Uri Treisman developed a collaborative approach to mathematics using challenging problems [8]. Although less familiar with his work when I initiated the Calculus Workshop, I have since searched the literature discovered many articles stemming from his work. This is an avenue ripe for exploration.

A large literature of Calculus problems has been published over the years. I have reviewed various texts (c.f. [7], [2], [5], [9], [1]) for raw material and modified problems to meet the needs of the Workshop. Additionally, there have been many studies done involving calculus workshops at various institutions pursuing a variety of research questions (c.f. [6], [4], [3], for example) which I have tried to apply when possible to my own research. One key difference between the workshops I have found in the literature and my own is that although I try to select problems that reflect the area being covered in the lecture, the workshops are not intended to directly supplement the class material as a means in itself, but to provide an avenue for students to explore more challenging problems in a self guided group environment.

3 A Sample Problem – Converging Circles

Historically, internalizing and truly understanding the concept of convergence has presented students and instructors alike with a substantial challenge. I have found this to be true whether convergence is viewed from the perspective of improper integrals, sequences, or series, the last of which students generally regard as the most difficult. I have found the converging circles problem, described below, useful for many reasons: It presents a visual representation of a convergent series, it can be solved using geometric methods, the "answer" can be readily grasped from the diagram with almost no work and when written numerically, the series telescopes conveniently and can be calculated easily.

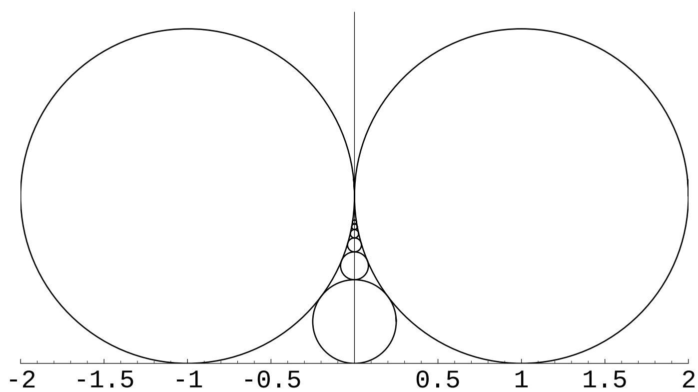

Figure 1: Problem: Determine the series of diameters of the central circles

The problem is as follows: we start with two unit circles, tangent to each other and a line (for convenience, we may regard the line as the x–axis, though this is by no means necessary). We then create a sequence of circles, circle c_1 tangent to both of the original unit circles and the x–axis, c_2 tangent to both circles and c_1, etc., as illustrated in figure 1 above. We form the series by taking the sum of the diameters of the circles c_1, c_2, c_3, ...

$$\sum_{n=1}^{\infty} c_n$$

Students generally attack the problem using right triangles, trying to incorporate the known quantities (viz. the radius of the large circle) as sides of the triangle. Some initial attempts are illustrated below in figure 2.

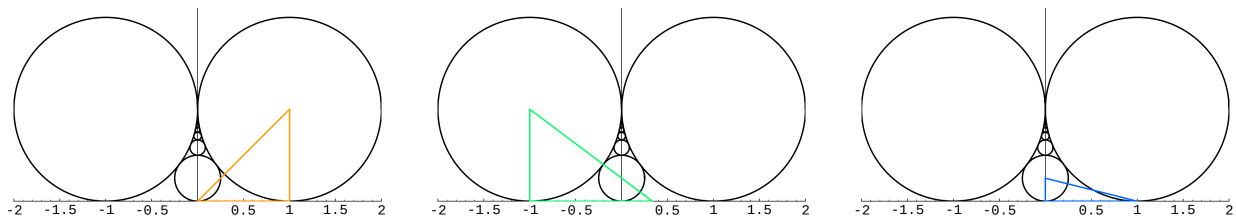

Figure 2: Typical student strategies

Although these represent a good start, the students quickly realize that they have not incorporated the radii of the central circle, c_1, and one of the large circles in their triangle. Eventually, one group comes up with something resembling figure 3, at which point we pause and have that group present their idea on the board so that everyone is on the same page.

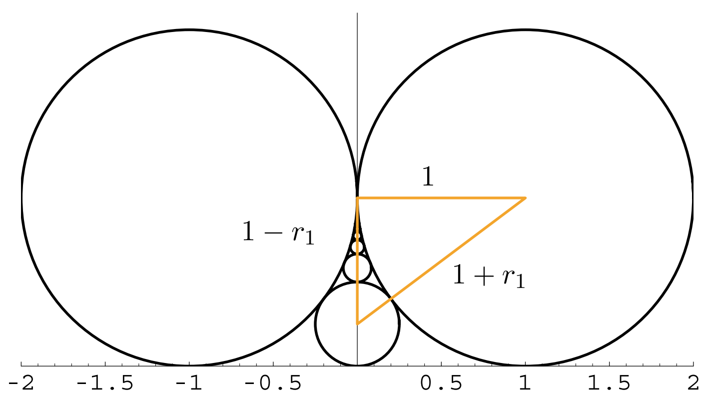

Figure 3: Diagram for a possible solution

From here the students quickly see the Pythagorean relationship and solve

$$(1 - r_1)^2 + 1^2 = (1 + r_1)^2$$

yielding $r_1 = 1/4$ and $c_1 = 1/2$, and continue finding the diameters recursively,

$$(1 - 2r_1 - r_2)^2 + 1^2 = (1 + r_2)^2$$

as illustrated below in figure 4. Substituting in $r_1 = 1/4$ yields $r_2 = 1/12$ and $c_2 = 1/6$.

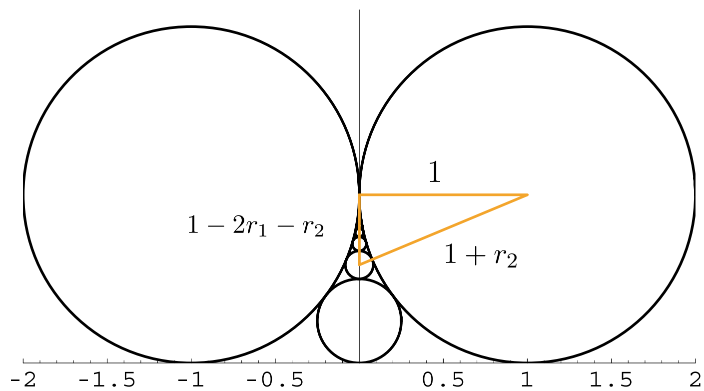

Figure 4: Second iteration of solution

With a procedure in hand, the groups set about racing each other to find the numerical values of the diameters, eventually coming up with the series,

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots$$

With some gentle coaxing find the relations

$$d_n = \frac{1}{n(n+1)}$$

and

$$d_n = \frac{1}{n} - \frac{1}{n+1}$$

at which point I have them work out the telescoping series

$$\sum_{n=1}^{\infty} d_n = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) = 1.$$

4 Observations and Impressions

The students are generally quite surprised by this point to see how different methods of calculating the sum of series relate to each other, and their relative levels of difficulty. As far as providing guidance for the students, there is a fine line differentiating offering too much help and letting the students languish. I have found that with these difficult problems, it is more useful to err on the side of less guidance. By providing positive feedback when students are heading down a path that should yield fruitful results, rather than guiding them down that path, the students take command of their learning and develop mathematical and problem solving self–sufficiency. Additionally, by having the students present the breakthroughs to the rest of the class as they occur, the students not only get a chance to express original mathematical ideas to their peers, but help to maintain the pace of the workshop. I encourage the rest of the groups to ask questions of the presenter, hence further encouraging self-sufficiency.

One of my most interesting observations involves a small selection of students who seemed completely uninterested in the lecture and classroom activities, and in fact were doing quite poorly in the class (some having failed the class previously), but came to life in the workshop. Their performance in the class improved, but more importantly, they suddenly seemed to enjoy mathematics and the problem solving process in general. One student in particular changed his major to mathematics so he could become a high school mathematics teacher. He is currently enrolled in my Linear Algebra class and remains excited about pursuing a career in mathematics education. I have also had several students become minors in mathematics as a direct result of their participation in the workshop.

5 Areas for Future Study

I would like to eventually institutionalize this model for use at CSUMB, so that it is offered every semester whether or not I teach Calculus. This presents a variety of challenges including preparing the instructor for the specific needs of the workshop as well as the omnipresent funding issues. One tack I have taken to meet these challenges is to adopt existing classes to this style of pedagogy where appropriate. Every Calculus student at CSUMB must complete a one credit lab concurrently with the lecture. I rewrote the material for the second semester Calculus Labs incorporating many of the concepts I learned from the workshop. The new labs have proven to be very successful with respect to both student learning and student affect. I plan to follow suit with the first semester Calculus labs as well.

A second area which seems like a very promising research question relates to the types of students described at the end of section 4. Indeed, why are some students engaged in this setting but not in the classroom? Which aspects of the workshop environment can be incorporated into the classroom to stimulate those students?

References

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